سكشن نظم

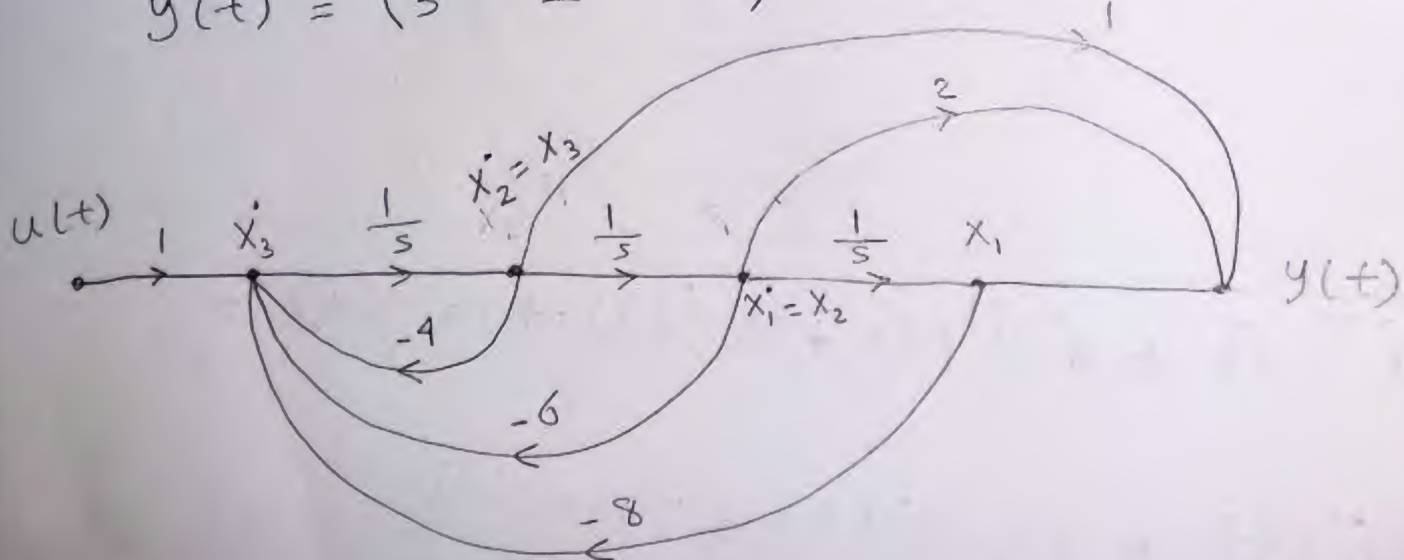
c) $\dfrac{Y(s)}{U(s)} = \dfrac{s^2 + 2s + 3}{s^3 + 4s^2 + 6s + 8}$

$$\dot{x}(t) = A_{3\times3}\, x(t)_{3\times1} + B_{3\times1}\, u(t)$$

$$y(t) = C_{1\times3}\, x(t) + D\, u(t)$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

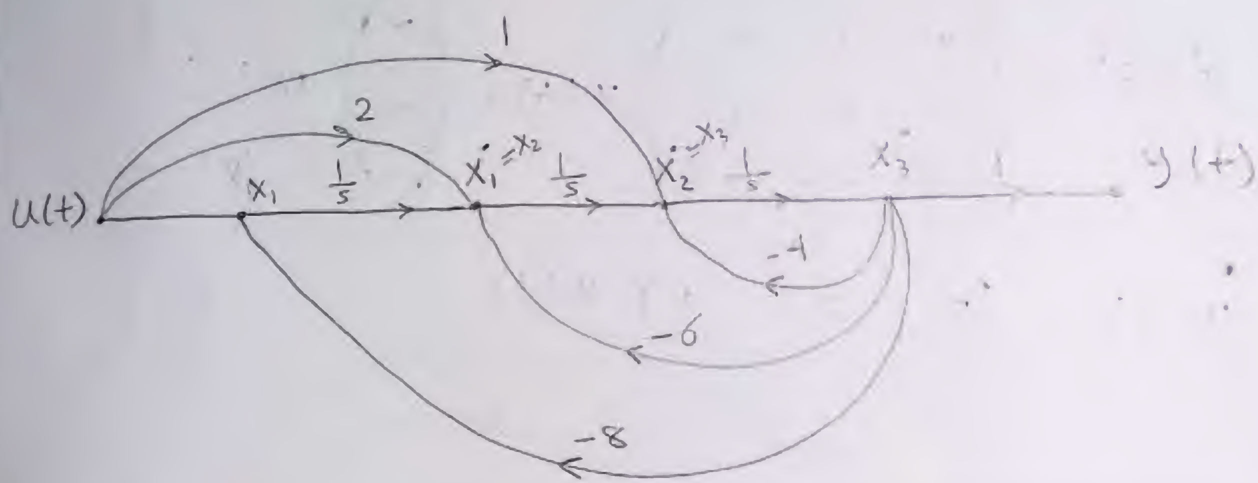
$$y(t) = \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} x(t)$$



$\boxed{1}$ Last sec.

*
$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{pmatrix} 0 & 0 & -8 \\ 1 & 0 & -6 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$



* $\dddot{y}(t) + 6\ddot{y}(t) + 8\dot{y}(t) + 10\,y(t) =$

$20\ddot{u}(t) + 25\dot{u}(t) + 30\,u(t)$

$\boxed{2}$ Last sec.

* $\dddot{y}(t) + 6\ddot{y}(t) + 8\dot{y}(t) + 10\,y(t) =$

$20\,\ddot{u}(t) + 25\,\dot{u}(t) + 30\,u(t)$

• Laplace

$s^3 Y(s) + 6s^2 Y(s) + 8s\,Y(s) + 10\,Y(s)$

$= 20\,s^2 U(s) + 25\,s\,U(s) + 30\,U(s)$

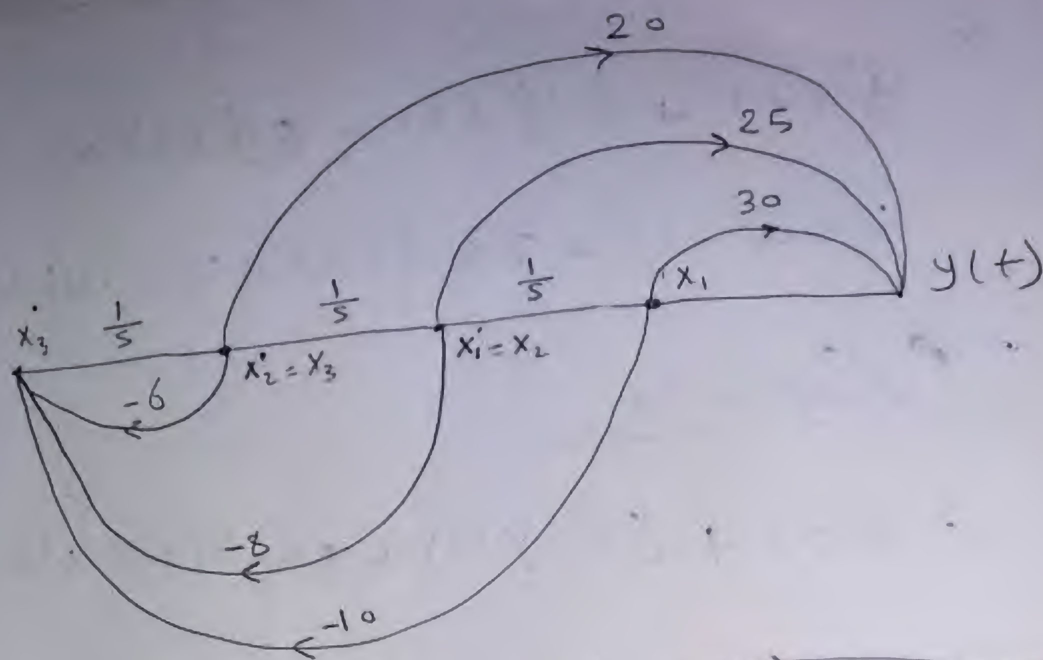
$$Y(s)\left[s^3 + 6s^2 + 8s + 10\right] = U(s)\left[20\,s^2 + 25\,s + 30\right]$$

$$\frac{Y(s)}{U(s)} = \frac{20\,s^2 + 25\,s + 30}{s^3 + 6s^2 + 8s + 10}$$

$$y(t) = \begin{pmatrix} 30 & 25 & 20 \end{pmatrix} x(t)$$

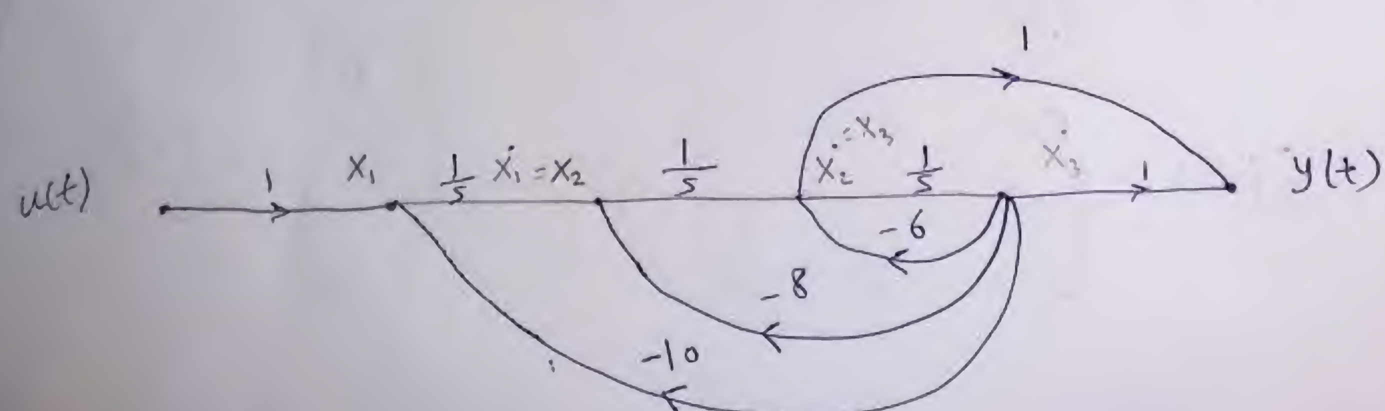
$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -8 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$\boxed{3}$ Last sec.



$$\dot{x}(t) = \begin{pmatrix} 0 & 0 & -10 \\ 1 & 0 & -8 \\ 0 & 1 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 30 \\ 25 \\ 20 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$



$\boxed{4}$ Last section

$$\boxed{2}\quad \dot{x}(t) = \underbrace{\begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix}}_{A} x(t) + \underbrace{\begin{pmatrix} 2 \\ 4 \end{pmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{pmatrix} 1 & 2 \end{pmatrix}}_{C} x(t)$$

Sol

$$\frac{Y(s)}{U(s)} = C\,(sI - A)^{-1} B + D$$

$$\Big[s \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{مصفوفة الوحدة}} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \Big]^{-1} = \left[\begin{pmatrix} s+5 & 1 \\ -3 & s+1 \end{pmatrix} \right]^{-1}$$

$$= \frac{1}{(s+5)(s+1)\ 3} \begin{pmatrix} s+1 & -1 \\ 3 & s+5 \end{pmatrix}$$

$$\frac{\begin{pmatrix} 1 & 2 \end{pmatrix}}{s^2 + 6s + 8} \begin{pmatrix} s+1 & -1 \\ 3 & s+5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{Y(s)}{U(s)}$$

$$= \frac{1}{s^2 + 6s + 8} \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2s - 2 \\ 4s + 26 \end{pmatrix}$$

$\boxed{5}$ Last sec.

$\cancel{X(s)}$ $\frac{Y(s)}{U(s)} = \frac{2s - 2 + 8s + 52}{s^2 + 6s + 8}$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{10s + 50}{s^2 + 6s + 8}}$$

$\boxed{4}$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ +3 & -2 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} U(t)$$

$$x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad U(t) \longrightarrow \text{unit step}$$

$$x(t) = \phi(t) \cdot X(0) + \int_0^t \phi(t-\tau) B\, u(\tau)\, d\tau$$

$\boxed{\text{sol}}$

$$\phi(t) = \mathcal{L}^{-1} (sI - A)^{-1}$$

$$\phi(t) = \mathcal{L}^{-1} \begin{pmatrix} s & -1 \\ -3 & s+2 \end{pmatrix}^{-1}$$

$$= \mathcal{L}^{-1} \left[\frac{1}{s^2 + 2s - 3} \cdot \begin{pmatrix} s+2 & 1 \\ +3 & s \end{pmatrix} \right]$$

Last. sec $\boxed{6}$

$$= \mathcal{L}^{-1}\begin{pmatrix} \dfrac{s+2}{s^2+2s-3} & \dfrac{1}{s^2+2s-3} \\ \dfrac{-3}{s^2+2s-3} & \dfrac{s}{s^2+2s-3} \end{pmatrix}.$$

$$= \mathcal{L}^{-1}\begin{pmatrix} \dfrac{\frac{1}{2}}{s+3} + \dfrac{\frac{3}{4}}{s-1} & \dfrac{A_3}{s+3} + \dfrac{A_4}{s-1} \\ \dfrac{A_1}{s+3} + \dfrac{A_2}{s-1} & \dfrac{A_5}{s+3} + \dfrac{A_6}{s-1} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} e^{-3t} + \frac{3}{4} e^{t} & A_3 e^{-3t} + A_4 e^{t} \\ A_1 e^{-3t} + A_2 e^{t} & A_5 e^{-3t} + A_6 e^{t} \end{pmatrix}$$

$$\boxed{\phi(t) * x(0) = \begin{pmatrix} B_1 e^{-3t} + B_2 e^{t} \\ B_3 e^{-3t} + B_4 e^{t} \end{pmatrix}}$$

الثوابت دي تساوي أرقام لا تكتب كثوابت يعني .

$$\int_0^t \begin{pmatrix} A_3 e^{-3-\tau} + A_4 e^{(t-\tau)} \\ A_5 e^{-3-\tau} + A_6 e^{t-\tau} \end{pmatrix} d\tau$$

(2)

$\boxed{7}$ *Last sec.*

$$= \begin{pmatrix} \frac{A_3}{3} - \frac{A_3}{3} e^{-3t} - \underline{\underline{A_4}} + A_4 e^{t} \\ \frac{A_5}{3} - \frac{A_5}{3} e^{-3t} - A_6 + A_6 e^{t} \end{pmatrix} \longrightarrow (2)$$

$x(t) = (1) + (2)$

$\boxed{6}$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -8 & -6 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 8 & 1 \end{pmatrix} x(t)$$

<u>Sol</u>

$$|sI - A| = \left| \begin{pmatrix} s & -1 \\ 8 & s+6 \end{pmatrix} \right|$$

$$= s^2 + 6s + 8 = 0$$

$$(s+2)(s+4) = 0 \longrightarrow s = -2, \ s = -4$$

on L.H.S $\longrightarrow$ system stable.

$\boxed{8}$ Last sec.

$$\frac{X(s)}{U(s)} = \begin{pmatrix} 8 & 1 \end{pmatrix} \begin{pmatrix} s+6 & 1 \\ -8 & s \end{pmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} * \frac{1}{(s+2)(s+4)}$$

$$= \begin{pmatrix} 8 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ s \end{pmatrix} * \frac{1}{(s+2)(s+4)}$$

$$= \frac{s+8}{(s+2)(s+4)}$$

$$\phi(t) = \mathcal{L}^{-1} (sI - A)^{-1}$$

<u>أكل</u> ⁝

$\boxed{9}$ Last sec.